

Supplementary Materials: Reason-and-Execute Prompting: Enhancing MultiModal Large Language Models for Solving Geometry Questions

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A PROMPTING TEMPLATE.

According to the requirements of different prompting methods, we have designed RaE prompting templates, CoT prompting templates, and PAL prompting templates for MM-LLMs to solve geometry questions, which are shown in Tables 1, 2, and 3, respectively.

Table 1: Two examples of RaE prompting template

Let's use Python to solve geometry questions. Here are two examples of how to do it	
Q1: Given the images, in $\triangle ABC$, it is known that $\angle A=80^\circ$, $\angle B=60^\circ$, $DE\parallel BC$. Then, the measure of $\angle CED$ is ()	
<pre>#Reasoning: The question involves finding the degree measure of angle $\angle CED$. From the exercise text and the diagram, it can be inferred that solving this problem requires the use of the triangle angle sum theorem and corresponding angles postulate. Corresponding Angles Postulate: $\angle CED=180^\circ-\angle C$ Triangle Angle Sum Theorem: $\angle C=180^\circ-\angle A-\angle B$ Information to be obtained from question texts and images: $\angle A=80^\circ, \angle B=60^\circ$ The solution that needs to be converted into Python code is: $\angle A=80, \angle B=60, \angle C=180-\angle A-\angle B, \angle CED=180-\angle C$ # Executing: def solution(): """The solution that needs to be converted into Python code is: $\angle A=80, \angle B=60, \angle C=180-\angle A-\angle B, \angle CED=180-\angle C$ """ angle_A = 80 angle_B = 60 angle_C = 180 - angle_A - angle_B angle_CED = 180 - angle_C result = angle_CED return result</pre>	
Q2: "Given the images, with circle O, chord AB = 18, M is the midpoint of AB, and OM = 12, then the radius of circle O is ()"	
<pre>#reasoning: The question is to find the radius OB of the circle. It can be inferred from the exercise text and the image that solving this problem requires the Pythagorean theorem. Pythagorean theorem: $OB^2=OM^2+BM^2$ M is the midpoint of AB: $BM=0.5*AB$ Information to be obtained from question texts and images: $OM=12, AB=18$ The solution that needs to be converted into Python code are: $OM=12, AB=18, BM=0.5*AB, OB^2=OM^2+BM^2$ #Executing: import math def solution(): """The operations that need to be converted into Python code are: $OM=12, AB=18, BM=0.5*AB, OB^2=OM^2+BM^2$""" Line_AB = 18 Line_OM = 12 Line_BM = Line_AB / 2 Line_OB = math.sqrt(Line_OM * Line_OM + Line_BM * Line_BM) result = Line_OB return result</pre>	

Table 2: Two examples of CoT prompting template

Step-by-step solving geometry questions. Here are two examples of how to do it	
Q1: As shown in the figure, the line EF intersects AB at point E, intersects CD at point F, EG bisects $\angle BEF$, intersects CD at point G, $\angle 1=50^\circ$, then $\angle 2$ is equal to ()	
<pre># Step-by-step solving geometry question: In this geometry problem, we have two parallel lines AB and CD, with a transversal EF intersecting AB at E and CD at F. A line segment EG bisects angle BEF and intersects CD at G. We are given that angle 1 ($\angle 1$) is 50°, and we need to find the measure of angle 2 ($\angle 2$). The known variables are that AB is parallel to CD, $\angle 1$ is 50°, and EG bisects $\angle BEF$. The unknown variable is the measure of $\angle 2$. Since AB//CD and EF is a transversal, we can use properties of alternate interior angles, corresponding angles, and the angle bisector principle to find the unknown angle. First, we need to establish the relationship between the given angles and $\angle 2$. Then, by understanding that EG bisects $\angle BEF$, we can infer relationships between the angles formed. Performing Calculations: 1. Since AB//CD and EF is a transversal, $\angle BEF$ and the angle formed by EF and CD (let's call it $\angle EFD$ for clarification) are supplementary because they form a linear pair. Knowing $\angle 1$ is 50° and it's given as an alternate interior angle to $\angle EFD$ (since AB//CD and EF is a transversal), we can establish that $\angle EFD$ also equals 50°. 2. Since EG bisects $\angle BEF$, it divides it into two equal parts. Let's denote the measure of each part as x. Therefore, $\angle BEG = x$ and $\angle GEF = x$. 3. We can find x by understanding that $\angle BEF + \angle EFD = 180^\circ$ (linear pair). However, since $\angle 1 = 50^\circ$ and it corresponds to $\angle EFD$, we know $\angle EFD$ is also 50°. This helps in establishing the entire angle $\angle BEF = 180^\circ - 50^\circ = 130^\circ$. Since EG bisects $\angle BEF$, $x = 130^\circ / 2 = 65^\circ$. 4. Therefore, $\angle BEG = x = 65^\circ$. Since $\angle 2$ is an alternate interior angle to $\angle BEG$ (with AB//CD and EG as the transversal), $\angle 2$ also equals 65° by the property of alternate interior angles.</pre>	
This is our answer:65	
Q2: As shown in the figure, the radii of two concentric circles are 4cm and 5cm respectively. If one chord AB of the larger circle is tangent to the smaller circle, then the length of chord AB is ()	
<pre># Step-by-step solving geometry question: In this problem, we need to determine the length of a chord that is tangent to two intersecting circles, one larger and one smaller. We are dealing with a geometric problem involving the Pythagorean theorem and properties of triangles. We know the diameter OC of the larger circle is 4 centimeters, and the radius OA of the smaller circle is 5 centimeters. Given that OC⊥AB and AC=BC=1/2AB, we need to find the length of the chord AB. We will use the Pythagorean theorem to solve this problem. By constructing a right triangle and utilizing the Pythagorean theorem, we can find the length of the chord AB. Connect OC and AO. Chord AB of the larger circle is tangent to the smaller circle. Since OC⊥AB, AC=BC=AB/2. Let's calculate the length of AB. Since OA=5cm and OC=4cm, in the right triangle AOC, $AC=\sqrt{(OA^2-OC^2)}=3\text{cm}$. $AB=2*AC=6\text{cm}$</pre>	
This is our answer: 6	

Table 3: Two examples of PAL prompting template

Step-by-step solving geometry questions. Here are two examples of how to do it	
Q1: Given the images, in $\triangle ABC$, it is known that $\angle A=80^\circ$, $\angle B=60^\circ$, $DE\parallel BC$. Then, the measure of $\angle CED$ is ()	
<pre># solution in Python: def solution(): """Given the images, in $\triangle ABC$, it is known that $\angle A=80^\circ$, $\angle B=60^\circ$, $DE\parallel BC$. Then, the measure of $\angle CED$ is ()""" angle_A = 80 angle_B = 60 angle_C = 180 - angle_A - angle_B angle_CED = 180 - angle_C result = angle_CED return result</pre>	
Q2: In the images, where AB is parallel to CD, the line EF intersects AB at point E and intersects CD at point F. Line EG bisects angle BEF, intersecting CD at point G. Given that angle 1 is 50° , the measure of angle 2 is ()	
<pre># solution in Python: def solution(): """In the images, where AB is parallel to CD, the line EF intersects AB at point E and intersects CD at point F. Line EG bisects angle BEF, intersecting CD at point G. Given that angle 1 is 50°, the measure of angle 2 is ()""" angle_1 = 50 angle_BEf = 180 - angle_1 angle_BEg = angle_BEf / 2 angle_2 = angle_BEg result = angle_2 return result</pre>	

Table 4: Domain knowledge statistics for GeoQA+ dataset (Partial)

Domain Knowledge	Quantity
Properties of isosceles triangles	37
Properties of the vertical bisector of a line segment	48
Properties of a right triangle	4
Properties of equilateral triangles	8
Polygon inner and outer angles	17
Triangle median line theorem	110
Properties of parallelograms	179
External angle properties of triangles	29
Triangle inner angles and theorem	57
Circular angle theorem	79
Outer circle and center of a triangle	9
Properties of inscribed quadrilaterals in circles	4
Properties of tangents	40
Calculation of Cones	19
Relationship between center angle, arc, and chord	2
Inscribed circle and inner circle of a triangle	2
Properties of parallel lines	315
Right-angled triangle with a 30-degree angle	8
Determination and properties of parallel lines	13
Determination of parallel lines	3
Properties of congruent triangles	38
Determination and properties of congruent triangles	18
Properties of the angular bisector	39
Definition of acute trigonometric function	4
Solving right-angled triangles	12
Trigonometric function value for special angles	1
Regular polygons and circles	3
Median line on the hypotenuse of a right triangle	19
Property of diamond	79
Properties of rectangles	66
Properties of a square	6
Definition of angular bisector	2
Direction angle	6
Drawing - Basic Drawing	2
Nature of translation	4
Vertical line	12
To vertex angle and adjacent complementary angle	14
Shortest vertical line segment	1
Congruent graphics	2
Calculation of angles	6
Application of Pythagorean theorem	14
Area of triangle	275
Trapezoid	4
Mixup	1273

Table 5: Domain knowledge statistics for GeoQA dataset (Partial)

Domain Knowledge	Quantity
Parallel lines	104
Isosceles triangle	23
Adjacent complementary angles	10
Polygon	16
Right triangle	30
Cone calculations	48
Rotation	21
Similar triangles	287
Median in a triangle	22
Tangent	108
Sum of angles in a triangle	24
Perpendicular bisector	8
Central angle	245
Translation	10
Rectangle	8
Pythagorean theorem	14
Symmetry	17
Interior alternate angles	8
Angle bisector	31
Parallelogram	109
Corresponding angles	3
Equilateral triangle	6
Exterior angles of a triangle	28
Direction angle	9
Perpendicular	4
Perpendicular bisector theorem	8
Rhombus	15
Distance	55
Similarity	10
Square	6
Central angle of a circle	15
Sector area	9
Cylinder calculations	1
Trigonometric functions	46
Solid figures	4
Circumscribed quadrilateral	40
Corresponding angles	7
Dial angle	1
Three views	5
Mixup	3595

B DOMAIN KNOWLEDGE STATISTICS

To analyze the impact of different amounts of domain knowledge on MM-LLMs solving geometry questions, we counted the number of questions involved in different domain knowledge in the GeoQA+ and GeoQA datasets, as shown in Tables 4 and 5, respectively.